Adaptive Probabilistic Forecasting for Wind Energy Based on Generalised Logit Transformation and Bayesian Method

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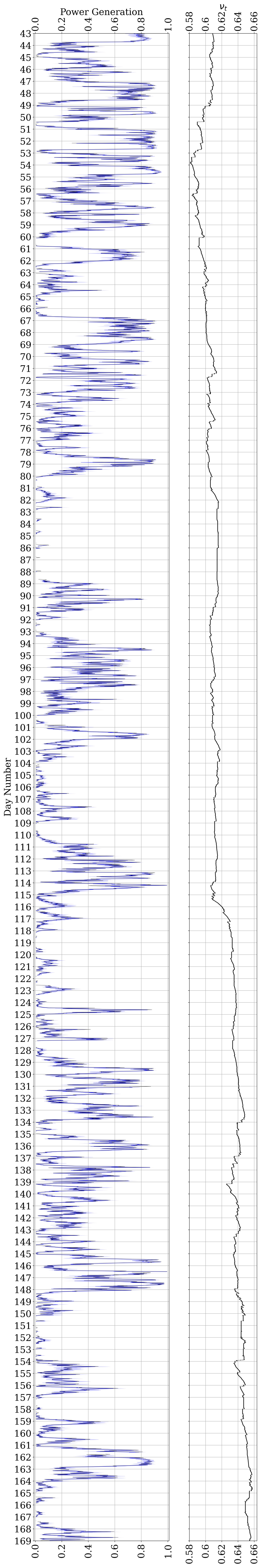

Figure 1: A case study of probabilistic forecasts is presented for a single wind farm using the Bayes-ν method. The observed data is represented by a dark line, the 50% quantile forecasts by a blue line, and the forecast interval, covering the 25% to 75% quantiles, is depicted as a blue patch. The variation of ν_t is shown alongside the forecast plot.

Background

Wind power plays an increasingly significant role in achieving the 2050 Net Zero Strategy. Accurately forecasting wind power generation is one key demand for the stable and controllable integration of renewable energy into existing grid operations. We propose an adaptive probabilistic forecasting method with 30-minutes resolution that combines the Generalised Logit Transformation with a Bayesian approach.

Highlights

- A **novel adaptive mechanism** for updating the shape parameter in Generalised Logit Transformation is introduced.
- An **extensive case study** of data over 100 wind farms ranging four years in the UK using four years of data.
- The **robustness** of the proposed methods is demonstrated through rank and sensitivity analysis, highlighting their reliability and consistency under varying conditions.

Proposed Methods

Generalised Logit Transformation definition:

$$Y := L_\nu(X) = \ln\frac{X^\nu}{1-X^\nu} \qquad X\in(0,1). \tag{1}$$

Auto-Regressive Model for wind power time series data under Generalised Logit Transformation:

$$L_\nu(x_t) = \theta_0+\theta_1 L_\nu(x_{t-1})+\theta_2 L_\nu(x_{t-2})+...+\theta_p L_\nu(x_{t-p})+z_t. \tag{2}$$

Likelihood for the observed wind power data:

$$p(\mathbf{x}_{t+M}|\mathbf{X}_{t+M,B},\boldsymbol{\theta}_t,\sigma^2_{z,t},\nu_t) = (2\pi\sigma^2_{z,t})^{-\frac{t+M-p}{2}}\prod_{i=p+1}^{t+M}\frac{\nu_t}{x_i(1-x_i^{\nu_t})} \exp\left\{-\frac{\|L_{\nu_t}(\mathbf{x}_{t+M})-L_{\nu_t}(\mathbf{X}_{t+M,B})^\top\boldsymbol{\theta}_t\|^2}{2\sigma^2_{z,t}}\right\}. \tag{3}$$

Hierarchical structure of proposed Bayesian method:

$$p(\tilde{\mathbf{y}}_{t+M}|\tilde{\mathbf{Y}}_{t+M,B},\boldsymbol{\theta}_t,\sigma_{z,t}) = \prod_{i=t+1}^{t+M} p(y_i|\mathbf{y}_{i,B},\boldsymbol{\theta}_t,\sigma_{z,t}),$$
$$\boldsymbol{\theta} = [\theta_0,...,\theta_p]^\top \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{\theta}},\Sigma_{\boldsymbol{\theta}}),$$
$$\sigma^{-2}_{z,t} \sim \mathcal{G}(\alpha,\beta). \tag{4}$$

Adaptive update for shape parameter ν:

$$\tilde{L}(\nu) := -\ln p(\tilde{\mathbf{x}}_{t+M}|\tilde{\mathbf{X}}_{t+M,B},\boldsymbol{\theta}^*_{t+M},\sigma^{*,2}_{z,t+M},\nu),$$
$$L^*(\nu) := -\ln p(\mathbf{y}_\nu|\mathbf{Y}_{B,\nu},\boldsymbol{\theta}^*_{t+M},\sigma^{*,2}_{z,t+M}),$$
$$\hat{\nu}_{t+M} = \arg\min_\nu\left\{L^*(\nu)+\tilde{L}(\nu)\right\},$$
$$\nu^*_{t+M} = (1-\gamma)\nu_{t+M}+\gamma\hat{\nu}_{t+M}. \tag{5}$$

$\mathbf{y}_\nu$ and $\mathbf{Y}_{B,\nu}$ are reconstructed data recovered from model covariance $\Sigma_{\boldsymbol{\theta}}$ using the Cholesky decomposition.

References

[1] Amandine Pierrot and Pierre Pinson. "Adaptive Generalized Logit-Normal Distributions for Wind Power Short-Term Forecasting". In: *2021 IEEE Madrid PowerTech*. 2021, pp. 1–6. DOI: 10.1109/PowerTech46648.2021.9494900.

[2] Pierre Pinson and Henrik Madsen. "Adaptive Modelling and Forecasting of Offshore Wind Power Fluctuations with Markov-Switching Autoregressive Models". In: *Journal of Forecasting* 31.4 (2012), pp. 281–313. ISSN: 1099-131X. DOI: 10.1002/for.1194.

Case Study

Seven methods have been implemented to compare against the proposed method. A total of 300+ test cases are evaluated.

Persistence The persistence method serves as a naive benchmark

AR-L Auto-Regressive model with standard logit data transformation

AR-L_ν AR with generalised logit data transformation

RLS AR with Recursive Least Square estimation [2]

NR AR with Newton-Raphson estimation [1]

Bayes AR with Bayesian estimation and fixed shape parameter ν

Bayes-ν AR with Bayesian estimation and varying shape parameter

Table 1: Performance metrics for averaged CRPS(%) and averaged Skill(%) over three test datasets.

	avg. CRPS (%)			avg. Skill (%)		
Test Dataset	**2021**	**2022**	**2023**	**2021**	**2022**	**2023**
Persistence	3.793	4.108	3.815	0.000	0.000	0.000
AR-L	3.684	3.987	3.676	2.975	3.106	3.751
AR-L_ν	3.671	3.976	3.664	3.339	3.422	4.096
RLS	3.676	3.996	3.667	3.217	2.950	4.004
NR	3.701	4.001	3.694	2.560	2.828	3.348
Bayes	3.656	3.977	3.655	3.744	3.432	4.329
Bayes-ν	**3.651**	**3.975**	**3.644**	**3.925**	**3.504**	**4.604**

Table 2: Frequency counts for different methods across ranks on CRPS result over all three test scenarios. Rank 1 counts the frequency that the method gets the best CRPS result.

Method	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7
Persistence	13	7	3	4	6	32	238
AR-L	4	41	48	52	95	43	20
AR-L_ν	31	44	**78**	80	54	16	0
RLS	1	7	51	73	85	84	2
NR	91	37	23	14	12	87	39
Bayes	34	**118**	**74**	49	23	5	0
Bayes-ν	**129**	49	26	31	28	36	4

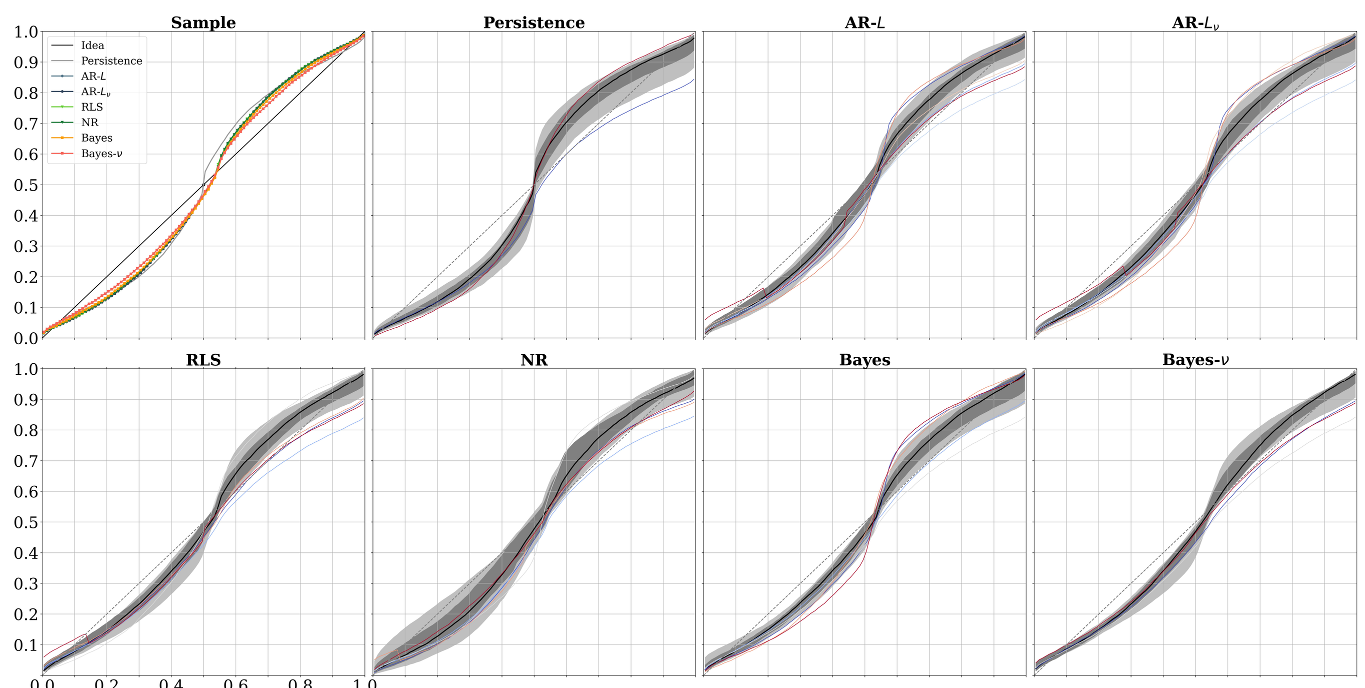

Figure 2: Reliability Diagram and Functional Reliability Diagram.

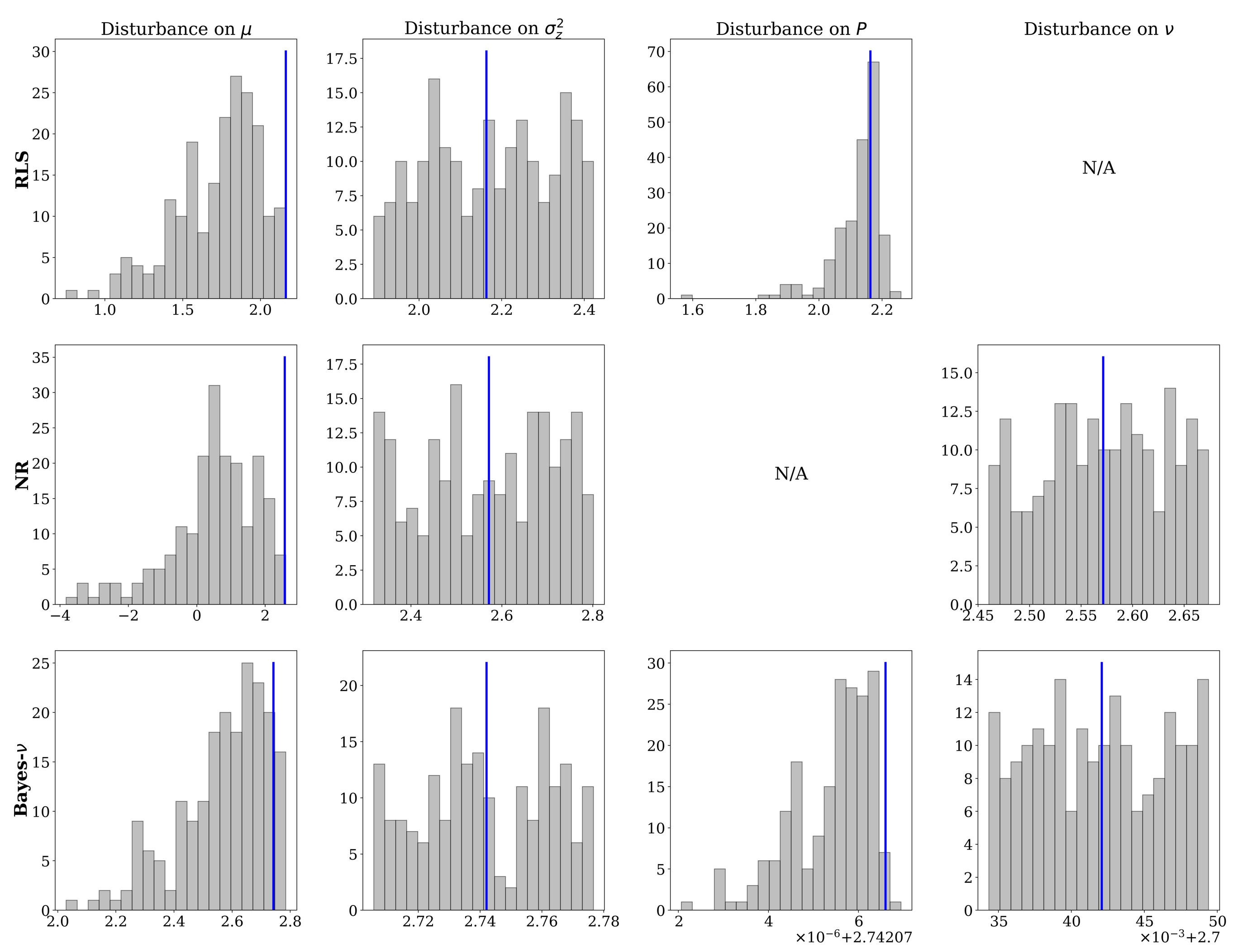

Figure 3: Sensitivity Analysis across the RLS, NR and Bayes-ν methods. Histogram of Skill Score are used to evaluate the robustness of the methods.

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